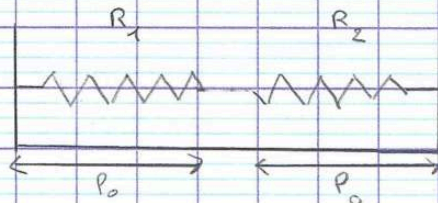
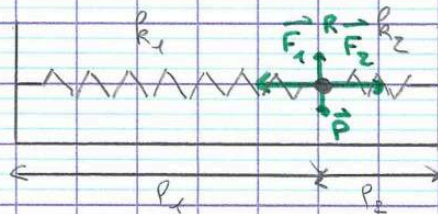


5 - Oscillations

Exercice 17: Ressorts horizontaux en opposition



Ressorts au repos



Ressorts en équilibre

- Ressorts tendus : $l_1 > l_0$ et $l_2 > l_0$
- $l_1 + l_2 = L$
- mouvement du mobile sans frottements

a) Bilan des forces

$$\vec{F}_1 + \vec{F}_2 + \vec{P} + \vec{R} = \vec{0} \quad \text{à l'équilibre}$$

Soit le référentiel Galiléen $(O, \vec{e}_x, \vec{e}_y, \vec{e}_z)$

$$\vec{F}_1 = -k_1 (l_1 - l_0) \vec{e}_x$$

$$\vec{P} = -mg \vec{e}_y$$

$$\vec{F}_2 = k_2 (l_2 - l_0) \vec{e}_x$$

$$\vec{R} = R \vec{e}_y$$

$$\begin{cases} -mg = R & (\vec{e}_y) \\ k_1 (l_1 - l_0) = k_2 (l_2 - l_0) & (\vec{e}_x) \end{cases}$$

à l'équilibre

$$\begin{cases} k_1 (l_1 - l_0) = k_2 (l_2 - l_0) \\ L = l_1 + l_2 \end{cases}$$

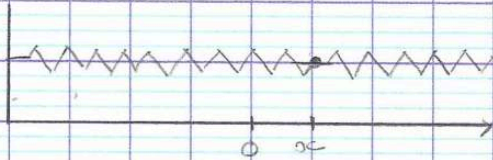
$$\begin{cases} l_1 = 0,436 \text{ m} \\ l_2 = 0,364 \text{ m} \end{cases}$$

$$k_1 l_1 - l_0 k_1 = k_2 l_2 - k_2 l_0$$

$$k_1 l_1 - k_2 l_2 = l_0 (k_1 - k_2)$$

$$2) \quad R = R_1 + R_2$$

$$x_{\max} = a = 6 \text{ cm}$$



a) Le ressort R_2 est-il encore tendu ?

$$l_2 - a > l_0$$

$$36,4 - 6 = 30,4 \text{ cm} > 80 \text{ cm}$$

R_2 est encore tendu

$$\sum \vec{F} = m \cdot \vec{a}$$

$$\vec{F}_1 + \vec{F}_2 + \vec{P} + \vec{R} = m \cdot \vec{a}$$

$$-k_1(l_1 - l_0)\vec{e}_x + k_2(l_2 - l_0)\vec{e}_x + R\vec{e}_y - mg\vec{e}_y = m a \vec{e}_x$$

$$\begin{cases} k_2(l_2 - l_0) - k_1(l_1 - l_0) = m \ddot{x} \\ R = -mg \end{cases}$$

b) Étude dynamique

Bilan des forces

$$\vec{OM} = x\vec{e}_x$$

$$\vec{v} = \dot{x}\vec{e}_x$$

$$\vec{a} = \ddot{x}\vec{e}_x$$

$$\vec{F}_1 = -k_1(l_1 + x - l_0)\vec{e}_x$$

$$\vec{F}_2 = k_2(l_2 - x - l_0)\vec{e}_x$$

$$\vec{P} = -mg\vec{e}_y$$

$$\vec{R} = R\vec{e}_y \quad (\text{pas de frottement})$$

PPD

$$\vec{F}_1 + \vec{F}_2 + \vec{P} + \vec{R} = m \vec{a}$$

$$\begin{cases} \vec{P} + \vec{R} = \vec{0} \quad e_y \\ -k_1(l_1 - l_0) - k_1x + k_2(l_2 - l_0) - k_2x = m\ddot{x} \end{cases}$$

équilibre

$$\begin{cases} \vec{P} + \vec{R} = \vec{0} \\ -k_1x - k_2x = m\ddot{x} \end{cases} \Leftrightarrow -R\ddot{x} = m\ddot{x}$$

$$x(t) = e^{Rt} \quad \dot{x}(t) = R e^{Rt} \quad \ddot{x} = R^2 e^{Rt}$$

$$e^{Rt}(R^2 + \omega^2) = 0$$

Solution du type $x(t) = a \cos(\omega t + \varphi)$
 période $T_0 = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{R}}$

On a $x(t) = C \cos(\omega t + \varphi)$

Utilisation des conditions initiales

$$x(t=0) = a = C \cos \varphi$$

$$v(t=0) = \dot{x}(t=0) = 0$$

$$\dot{x} = -\omega C \sin(\omega t + \varphi)$$

$$0 = -\omega C \sin \varphi$$

$$\begin{cases} a = C \cos \varphi \\ 0 = -\omega C \sin \varphi \end{cases}$$

$$C \neq 0 \Rightarrow \varphi = 0$$

$$\Rightarrow C = a$$

On a donc $-a \leq x(t) \leq a$

Vérifions que R , reste tendu

$$P_1 - a = 43,6 - 6 > 0$$

La force de rappel dérive de $\mathcal{E}_{pe}$

$$\vec{F} = R x \vec{e}_x$$

$$\vec{F} \cdot d\vec{OM} = -R x dx$$

$$dW = -R x dx$$

$$W = -\frac{R}{2} x^2 + k = -\mathcal{E}_{pe}$$

$$\Rightarrow \mathcal{E}_{pe} = R x^2 / 2 + k$$

si $x = 0$

$$\mathcal{E}_p = 0$$

$$\text{donc } \mathcal{E}_{pe} = + \frac{R}{2} x^2$$

$\Rightarrow \mathcal{E}_m = \mathcal{E}_c + \mathcal{E}_p$ est une constante du mouvement.

$$x(t) = a \cos \omega t$$

$$\mathcal{E}_p = \frac{1}{2} R (a \cos \omega t)^2 = \frac{1}{2} R a^2 \cos^2 \omega t$$

$$\mathcal{E}_c = \frac{1}{2} m v^2 = \frac{1}{2} m (-\omega a \sin \omega t)^2 = \frac{1}{2} m \omega^2 a^2 \sin^2 \omega t$$

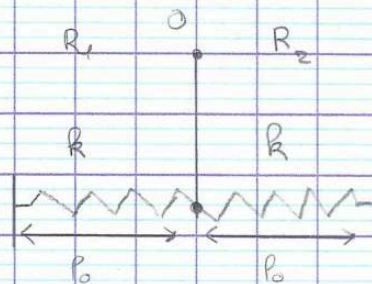
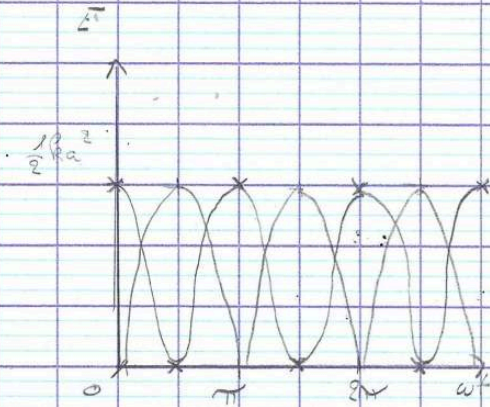
$$\mathcal{E}_m = \frac{1}{2} R a^2 \cos^2 \omega t + \frac{1}{2} m \omega^2 a^2 \sin^2 \omega t$$

OR on sait

$$\omega^2 = \frac{R}{m}$$

donc
$$\vec{L}_m = \frac{1}{2} k a^2 \cos^2 \omega t + \frac{1}{2} m \left(\frac{k}{m} \right) a^2 \sin^2 \omega t$$

$$= \frac{1}{2} k a^2 = \text{cte}$$



Exercice 18

%

R_1 et R_2 sont 2 ressorts identiques

(1) Partie cinématique (repère

(2) B.Pam des forces

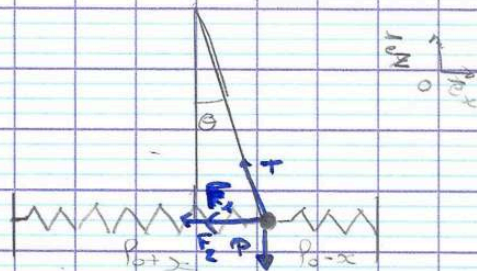
(3) PFD

PFD $\Delta \vec{F} = m \cdot \vec{a}$

$$\vec{P} + \vec{T} + \vec{F}_1 + \vec{F}_2 = m \cdot \vec{a}$$

équation vectorielle

$$\begin{cases} \vec{P} = -mg \vec{e}_y \\ \vec{T} = -T \sin \theta \vec{e}_x + T \cos \theta \vec{e}_y \\ \vec{F}_1 = -k(l_0 + x - l_0) \vec{e}_x \\ \vec{F}_2 = -k(l_0 + x - l_0) \vec{e}_x \end{cases}$$



Remarque

$$\vec{F} = -k |l_0 - l|$$

$$-mg \vec{e}_y - T \sin \theta \vec{e}_x + T \cos \theta \vec{e}_y - kx \vec{e}_x - kx \vec{e}_x = m \ddot{x} \vec{e}_x$$

$$\begin{cases} m \ddot{x} = -2kx - T \sin \theta \\ T \cos \theta = mg \end{cases} \Rightarrow \begin{cases} \ddot{x} = -\frac{2k}{m} x - T \sin \theta \\ T \cos \theta = mg \end{cases}$$

Ainsi: $\ddot{x} = -\frac{2k}{m}x - g \tan \theta$

or on a θ très petit: $\tan \theta \approx \theta$

$$\ddot{x} = -\frac{2k}{m}x - g\theta$$

$$= -\frac{2k}{m}x - g\frac{x}{p}$$

$$= \ddot{x} + \omega^2 x = 0$$

$$\omega^2 = \frac{2k}{m} + \frac{g}{p}$$

x = longueur de l'arc
 $= p\theta \Rightarrow \theta = \frac{x}{p}$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2k}{m} + \frac{g}{p}}}$$

Tg

$$T = \frac{2\pi}{\omega}$$

Exercice 19

$$m < M$$

Dans tout système physique, la quantité de mouvement est une grandeur conservée.

Au repos $\sum \vec{F} = 0$

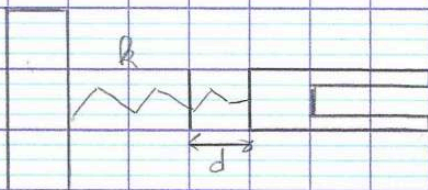
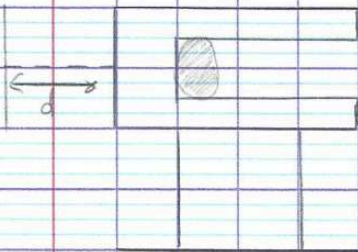
$$\vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} \Rightarrow \frac{d\vec{p}}{dt} = 0$$

$$P_{\text{final}} = P_{\text{initial}} = 0$$

$\Rightarrow p$ est une cte.

$$\vec{V} = -\frac{m}{M} \vec{v}$$

$$E_c(M) = \frac{1}{2} M \vec{V}^2 = \frac{1}{2} m v^2$$



Sur le schéma, le tir provoque une compression du ressort

$k_{\min} = k_m$ en fonction de v_0 , m , M et d ?

Deux approches possibles:

- PFD

- Énergie cinétique

Dans ce problème, la seule force à considérer est la force élastique

La force élastique dérive d'une énergie potentielle

⇒ la force est conservative.

⇒ E_m est une cte du mouvement

g : recul maximal

On s'intéresse ici au canon

$$E_{Ci} + E_{Pi} = E_{Cg} + E_{Pg}$$

$$\Rightarrow \frac{1}{2} MV^2 + 0 = 0 + \frac{1}{2} R d^2 \Rightarrow MV^2 = R d^2$$
$$R = \frac{MV^2}{d^2}$$

$$\Rightarrow R = \frac{m^2 v_0^2}{M d^2}$$

ainsi on doit avoir

$$R = R_m = \frac{m^2 v_0^2}{M d^2} \quad \text{si on veut une}$$

distance $\otimes$ courte il faut que $R \geq R_m$

b) Une énergie est absorbée lors du recul

donc $d' < d$

$$E_{Ci} + E_{Pi} = E_{Cg} + E_{Pg} + \omega$$

$$E_{Ci} + E_{Pi} = \frac{1}{2} M^2 + 0$$

$$E_{Cg} + E_{Pg} = 0 + \frac{1}{2} R d'^2 + \omega$$

$$\text{donc } \frac{1}{2} MV^2 = \frac{1}{2} R d'^2 + \omega$$

$$\text{donc } d' = d \sqrt{1 - \frac{2\omega}{R_m d^2}}$$

$$MV^2 = R d'^2 + 2\omega$$

$$\frac{MV^2 - 2\omega}{R} = d'^2$$

$$d'^2 = \frac{M}{R} \left(\frac{m}{M} v_0 \right)^2 - \frac{2\omega}{R}$$

$$= \frac{m^2 v_0^2}{R M} - \frac{2\omega}{R}$$

$$= d^2 - \frac{2\omega}{R}$$

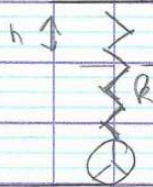
Exercice 20

(1) $M = 600 \text{ kg}$

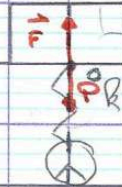
R

$h = 30 \text{ cm}$

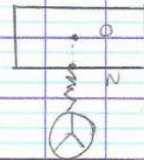
à vide
0 kg



à l'équilibre
600 kg



en mouvement



(a) Cinématique + Bilan des forces

$$\vec{F}_R = -R h \vec{e}_z$$

$$\vec{P} = M g \vec{e}_z$$

avec $\vec{F}_R + \vec{P} = 0$

$$M g = R h$$

$$\Rightarrow R = \frac{M g}{h} = 20\,000 \text{ N.m}^{-1}$$

$$\vec{ON} = z \vec{e}_z = z(t) \vec{e}_z$$

$$\vec{F}_R = -R(h+z) \vec{e}_z$$

$$\vec{P} = M g \vec{e}_z$$

$$\vec{F}_R + \vec{P} = M \vec{a}$$

$$-R(h+z) \vec{e}_z + M g \vec{e}_z = M \ddot{z} \vec{e}_z$$

$$-R h + M g - R z = M \ddot{z}$$

$$\Rightarrow M \ddot{z} + R z = 0$$

$$\Rightarrow \ddot{z} + \omega_0^2 z = 0$$

Équation d'un oscillateur

harmonique libre

$$T = \frac{2\pi}{\omega_0}$$

$$m = 300 \text{ kg}$$

$$m' = 5 \text{ kg}$$

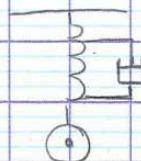
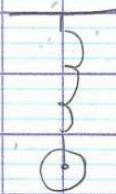
$$T' = 2\pi \sqrt{\frac{M+m'}{R}}$$

$$T' > T$$

(c) $\vec{f} = -b \vec{v}$ ($b > 0$)

PPD:

$$M g \vec{e}_z - R(h+z) \vec{e}_z - b \vec{v} = M \ddot{z} \vec{e}_z$$



$$\ddot{z} + \frac{b}{m} \dot{z} + \frac{k}{m} z = 0$$

$$\vec{g} = -b\vec{v}$$

$$2\lambda = \frac{b}{m} \quad \text{et} \quad \omega_0^2 = \frac{k}{m}$$

$$R^2 + 2\lambda R + \omega_0^2 = 0$$

$$\Delta = 4(\lambda^2 - \omega_0^2)$$

$$\Delta > 0 \Rightarrow 2 \text{ racines} \\ z = Ae^{R_1 t} + Be^{R_2 t}$$

apériodique

$$\Delta = 0 \Rightarrow 1 \text{ racine} \\ z(t) = e^{-\lambda t} (A + Bt)$$

critique

$$\Delta < 0 \Rightarrow 2 \text{ racines} \\ z = Ae^{R_1 t} + Be^{R_2 t}$$

pseudo périodique

Le régime est critique donc $\lambda = \omega_0$

$$\ddot{z} + 2\lambda \dot{z} + \omega_0^2 z = 0$$

$$z(t) = e^{-\lambda t} (A + Bt)$$

Pour de condition initiale

déterminons b

$$\lambda^2 = \omega_0^2 \Rightarrow \left(\frac{b}{2m}\right)^2 = \frac{k}{m} \Rightarrow b = \sqrt{4kM}$$

b) On rajoute la masse des passagers

$$\ddot{z} + \frac{b}{(m + \frac{m}{4})} \dot{z} + \frac{k}{(m + \frac{m}{4})} z = 0$$

$$z(t) = e^{Rt}$$

$$\Rightarrow R^2 + \frac{b}{(M + \frac{m}{4})} R + \frac{R}{(M + \frac{m}{4})} = 0$$

$$\Delta = \frac{b^2 - 4R(M + \frac{m}{4})}{(M + \frac{m}{4})^2} \Rightarrow \Delta < 0 \quad (\text{en remplaçant } b).$$

$$R_1 = \frac{1}{2} \left(-\left(\frac{b}{M + \frac{m}{4}}\right) + i\Omega \right) \quad \text{avec} \quad \Omega = \frac{\sqrt{RM}}{M + \frac{m}{4}}$$

$$R_2 = \frac{1}{2} \left(-\left(\frac{b}{M + \frac{m}{4}}\right) - i\Omega \right)$$

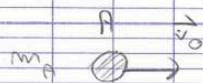
$$\begin{aligned} \text{ainsi: } z(t) &= A e^{R_1 t} + B e^{R_2 t} \\ &= e^{-\frac{bt}{2(M + \frac{m}{4})}} [A e^{i\Omega t} + B e^{-i\Omega t}] \\ &= e^{-\frac{bt}{2(M + \frac{m}{4})}} [D \cos(\Omega t + \phi)] \end{aligned}$$

on a la pseudo période

$$T = \frac{2\pi}{\Omega}$$

6 - Interaction à deux corps

Exercice 83

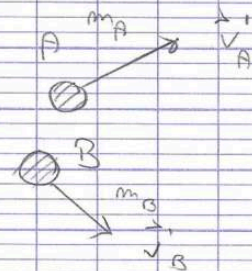


avant

Un choc est élastique si:

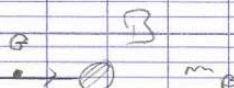
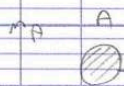
$$\vec{L}_c^A + \vec{L}_c^B = \vec{L}_c^{A'} + \vec{L}_c^{B'}$$

Poi de conservation des chocs

$$\vec{p}_A + \vec{p}_B = \vec{p}_A' + \vec{p}_B'$$


1) a) Dans le référentiel du laboratoire

$$\vec{V}_A = \frac{d\vec{OA}}{dt} = \vec{V}_0 \quad \vec{V}_B = \frac{d\vec{OB}}{dt} = \vec{0}$$



G centre de gravité ou centre de masse

$$m_A \vec{GA} + m_B \vec{GB} = \vec{0}$$

$$m_A (\vec{GO} + \vec{OA}) + m_B (\vec{GO} + \vec{OB}) = \vec{0}$$

$$\vec{OG} = \frac{m_A \vec{OA} + m_B \vec{OB}}{m_A + m_B}$$

$$\vec{V}_G = \frac{d\vec{OG}}{dt} = \frac{m_A \frac{d\vec{OA}}{dt} + m_B \frac{d\vec{OB}}{dt}}{m_A + m_B}$$

$$\vec{V}_G = \frac{m_A}{m_A + m_B} \vec{V}_0 \times \frac{1}{m_A} \quad \alpha = \frac{m_B}{m_A}$$

$$\vec{V}_G = \frac{1}{1+\alpha} \vec{V}_0$$

b) Soit G' le centre de masse du système A+B après le choc

$$m_A \vec{G'A} + m_B \vec{G'B} = \vec{0}$$

$$\Rightarrow (m_A + m_B) \vec{G'O} = m_A \vec{OA} + m_B \vec{OB}$$

$$\Rightarrow \vec{OG'} = \frac{m_A \vec{OA} + m_B \vec{OB}}{m_A + m_B}$$

$$\Rightarrow \vec{V}_{G'} = \frac{d\vec{OG'}}{dt} = \frac{m_A \frac{d\vec{OA}}{dt} + m_B \frac{d\vec{OB}}{dt}}{m_A + m_B}$$

$$\Rightarrow \vec{V}_{G'} = \frac{m_A \vec{V}_A + m_B \vec{V}_B}{m_A + m_B}$$

On sait que: $\vec{P}_{Ave} = \vec{P}_{Apc}$

donc $m_A \vec{v}_0 = m_A \vec{v}_A + m_B \vec{v}_B$

On en déduit que $\vec{v}_G = \frac{m_A \vec{v}_0}{m_A + m_B} = \frac{1}{1+\alpha} \vec{v}_0$

2) a) G est identique à G'. De plus le point G est animé d'un mouvement rectiligne uniforme. Donc un repère d'origine G est un repère Galiléen.

b) Soit $\vec{GA}$ et $\vec{GB}$ les vecteurs positions de A et B dans R^*

$\frac{d\vec{GA}}{dt} = \vec{v}_A^*$, $\frac{d\vec{GB}}{dt} = \vec{v}_B^*$

$$\frac{d(\vec{GO} + \vec{OA})}{dt} = -\vec{v}_G + \vec{v}_A = -\frac{1}{1+\alpha} \vec{v}_0 + \vec{v}_A$$

$$\frac{d(\vec{GO} + \vec{OB})}{dt} = -\vec{v}_G + \vec{v}_B = -\frac{1}{1+\alpha} \vec{v}_0 + \vec{v}_B$$

$$\vec{v}_A^* = -\frac{1}{1+\alpha} \vec{v}_0 + \vec{v}_A = \vec{v}_0 \left(1 - \frac{1}{1+\alpha} \right)$$

$$m_A \frac{\alpha}{1+\alpha} \vec{v}_0 = m_B \frac{\vec{v}_0}{1+\alpha}$$

$$\vec{v}_A^* = \frac{\alpha}{1+\alpha} \vec{v}_0$$

$$\frac{\vec{v}_0}{1+\alpha} (m_A \alpha - m_B)$$

0

$$\vec{v}_B^* = \frac{d(\vec{GO} + \vec{OB})}{dt} = -\vec{v}_G$$

$$\vec{p}_A^* = m_A \vec{v}_A^*$$

$$\vec{p}_B^* = m_B \vec{v}_B^*$$

$$\vec{L}_{CA}^* = \frac{1}{2} m_A v_A^{*2} = \frac{1}{2} m_A \left(\frac{\alpha}{1+\alpha} \right)^2 v_0^2$$

$$\vec{L}_{CB}^* = \frac{1}{2} m_B v_B^{*2} = \frac{1}{2} m_B \frac{v_0^2}{(1+\alpha)^2}$$

$$\vec{p}_A^* + \vec{p}_B^* = \vec{0}$$

$$\vec{L}_C^* = \frac{1}{2} m_A v_0^2 \left(\frac{\alpha}{1+\alpha} \right)^2 + \frac{1}{2} m_B v_0^2 \frac{\alpha}{(1+\alpha)^2} = \vec{L}_{CA} \frac{\alpha^2 + \alpha}{(1+\alpha)^2} = \frac{\alpha(1+\alpha)}{(1+\alpha)^2} \vec{L}_{CA}$$

$$\vec{L}_C^* = \frac{\alpha}{1+\alpha} \vec{L}_{CA}$$

3) Comparer $\vec{p}_A^*$ et $\vec{p}_B^*$

$$\vec{p}_A^* = m_A \vec{v}_A^*$$

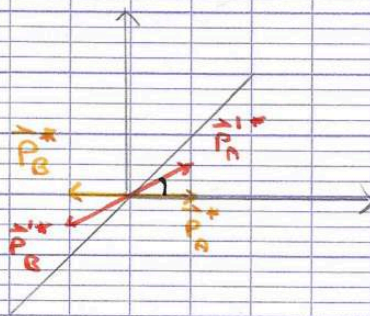
$$\vec{p}_B^* = m_B \vec{v}_B^*$$

Par conservation de la qte de movt dans R^*

$\vec{p}_{A+C}^* = \vec{p}_{A+B}^*$

$$\begin{aligned} \vec{p}_A + \vec{p}_B &= \vec{p}_C \\ \vec{p}_A + \vec{p}_B &= \vec{p}_C \\ \vec{p}_A + \vec{p}_B &= \vec{p}_C \end{aligned}$$

$$\vec{p}_A = -\vec{p}_B$$



$$\begin{aligned} \vec{p}_A &= \vec{p}_A' + \vec{p}_B' \\ \vec{p}_A &= \vec{p}_A' + \vec{p}_B' \\ \vec{p}_A &= \vec{p}_A' + \vec{p}_B' \\ \vec{p}_A &= \vec{p}_A' + \vec{p}_B' \end{aligned}$$

$$p_A'^2 = p_A^2 \Rightarrow p_A' = p_A$$

$$\Rightarrow p_A' = p_A = p_B' = p_B$$

3) c) Dans le repère du centre de masse

$$\|\vec{v}_A\| = cte$$

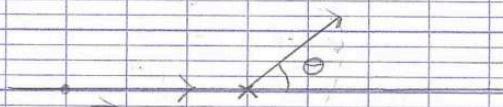
$$\vec{v}_A = \vec{v}_A'$$

$$v_A' = \frac{\alpha}{1+\alpha} v_0$$

$$v_B' = \frac{v_0}{1+\alpha}$$

$$\begin{cases} \|\vec{p}_A'\| = \|\vec{p}_A\| \\ \|\vec{p}_B'\| = \|\vec{p}_B\| \end{cases}$$

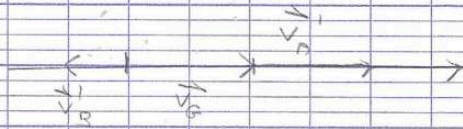
$$\Rightarrow v_A' = v_A \text{ et } v_B' = v_B$$



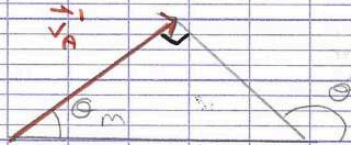
$$\vec{v}_A = \vec{v}_B$$

$$\text{OR } v_A = v_B \Rightarrow v_B' > v_A'$$

Si $\theta = 0^\circ$



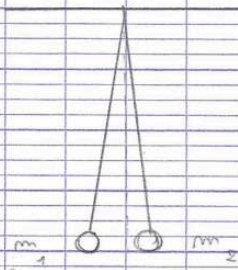
θ_m correspond à la situation où la direction du vecteur $\vec{v}_1$ est tangente au cercle



$$\sin \theta_m = \frac{\|\vec{v}_1\|}{\|\vec{v}_0\|} = \frac{\frac{\alpha}{1+\alpha} v_0}{\frac{v_0}{1+\alpha}} = \alpha$$

Exercice 25

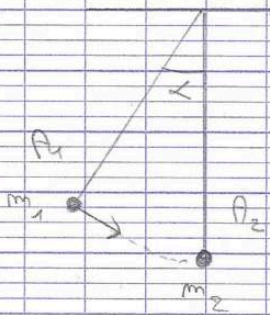
système



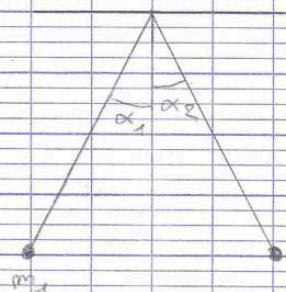
||

$m_1 + m_2$

avant



après



Conservation de l'é.c.

Soit $\vec{v}_i$ la vitesse de A_i et A_2 juste après le choc.

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$\Rightarrow v_1^2 = v_1'^2 + \frac{m_2}{m_1} v_2'^2$$

$$v_1^2 = v_1'^2 + v_2'^2$$

, conservation de la qdm

$$m_1 \vec{v}_1 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

Si $\vec{P}$ est conservé : $\frac{d\vec{P}}{dt} = \vec{0} = \vec{F}$ juste avant le choc on est proche de $\Sigma \vec{F} = \vec{0}$

$$\vec{P}_{\text{après}} = \vec{P}_1 + \vec{P}_2 = \vec{P}_{\text{av}}$$

$$m_1 \vec{v}_1 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

$$m_1 \vec{v}_1 = -m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

$$v_1 = -v_1' + x v_2'$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$v_1^2 = v_1'^2 + x v_2'^2$$

$$v_1' = x v_2' - v_1$$

$$v_1'^2 = (x v_2' - v_1)^2 + x v_2'^2$$

$$v_1'^2 = x^2 v_2'^2 + v_1^2 - 2x v_2' v_1 + x v_2'^2$$

$$0 = (x^2 + x) v_2'^2 - 2x v_2' v_1$$

comme $v_2' \neq 0$ (juste après le choc)

$$0 = (x^2 + x) v_2' - 2x v_1$$

$$= (x+1) v_2' - 2v_1$$

$$\text{d'où } v_2' = \frac{2v_1}{x+1}$$

D'autre part $v_1' = x \left(\frac{2v_1}{x+1} \right) - v_1$

$$v_1' = \frac{2xv_1 - v_1}{x+1}$$

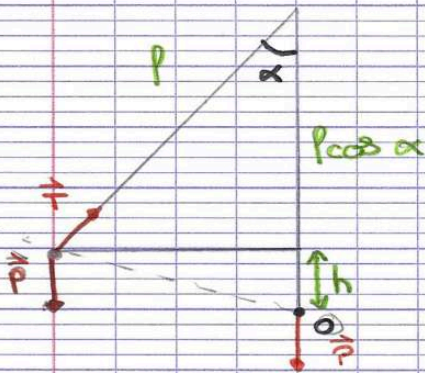
$$= \frac{(x-1)v_1}{x+1}$$

$$v_1' = \frac{x-1}{x+1} v_1$$

L'énergie mécanique du système est conservée car:
 le travail de la force $\vec{T}$ est nulle: $\vec{T} \perp \vec{v}_1$
 $dW_T = \vec{T} \cdot d\vec{OM}$ avec $d\vec{OM} \parallel \vec{v}$
 $\vec{T}$ est $\perp$ à $d\vec{OM}$
 et $\vec{p}$ dérive d'une énergie potentielle.

$$\bar{L}_m = \bar{L}_c + \bar{L}_p = cte$$

On sait que $\bar{L}_p = mgz$



$$\bar{L}_p = mgh$$

$$h = l - l \cos \alpha = l(1 - \cos \alpha)$$

$$\bar{L}_p = mgl(1 - \cos \alpha)$$

$$\bar{L}_m(t=0) = \bar{L}_m(\text{juste au choc})$$

$$0 + mgl(1 - \cos \alpha) = \frac{1}{2} m v_1^2 + 0$$

$$v_1^2 = 2gl(1 - \cos \alpha)$$

$$v_1 = (2gl(1 - \cos \alpha))^{1/2}$$

$$\left\{ \begin{array}{l} v_1' = \frac{x-1}{x+1} \sqrt{2gl(1 - \cos \alpha)} \\ v_2' = \frac{2}{x+1} \sqrt{2gl(1 - \cos \alpha)} \end{array} \right.$$

Juste après le choc $\bar{L}_m = \bar{L}_m(t)$

$$\frac{1}{2} m_1 v_1'^2 + 0 = 0 + m_2 gl(1 - \cos \alpha_1)$$

$$v_1'^2 = 2gl(1 - \cos \alpha_1)$$

$$v_1' = [2gl(1 - \cos \alpha_1)]^{1/2}$$

$$\text{donc } v_1'^2 = 2gl(1 - \cos \alpha_1) = \left(\frac{x-1}{x+1} \right)^2 (1 - \cos \alpha)$$

$$\cos \alpha_1 = 1 - \left(\frac{x-1}{x+1} \right)^2 (1 - \cos \alpha)$$

idem pour v_2

$$v_2'^2 = 2gl(1 - \cos \alpha_2)$$

$$2gl(1 - \cos \alpha_2) = \left(\frac{2}{x+1} \right)^2 2gl(1 - \cos \alpha)$$

$$\cos \alpha_2 = 1 - \frac{4}{(x+1)^2} (1 - \cos \alpha)$$

b) Le choc est ici (parfaitement) inélastique ou choc mou. Conservation de la qte de movt juste avant et juste après le choc.

$$m_1 \vec{v}_1 = (m_1 + m_2) \vec{v}_{12}'$$

$$v_1 = (1+x) \vec{v}_{12}'$$

$$v_1 = \sqrt{2gl(1 - \cos \alpha)}$$

$$L'_m = cte$$

$$\frac{1}{2} (m_1 + m_2) v_{12}'^2 + 0 = 0 + (m_1 + m_2) gl(1 - \cos \alpha)$$

$$v_{12}'^2 = 2gl(1 - \cos \alpha')$$

$$v_{12}'^2 = 2gl(1 - \cos \alpha') = \left(\frac{v_1}{1+x} \right)^2$$

$$2gl(1 - \cos \alpha') = \frac{2gl(1 - \cos \alpha)}{(1+x)^2}$$

$$\cos \alpha' = 1 - \frac{1}{(1+x)^2} (1 - \cos \alpha)$$

$$2) \alpha = 60^\circ$$

choc élastique $x = \frac{m_2}{m_1}$

$$S: \alpha_1 = \alpha_2 \Rightarrow \cos \alpha_1 = \cos \alpha_2$$

$$99 \times 15$$

$$1 \times 120$$

$$99 \times 15$$

$$1 \times 5h 30$$

$$99 \times 20$$

$$1 \times 5h$$

$$99 \times 27$$

$$1 \times 5h$$

$$99 \times 60$$

$$1 + \left(\frac{x-1}{x+1} \right)^2 (1 - \cos \alpha) = 1 - \frac{4}{(x+1)^2} (1 - \cos \alpha)$$

b) $\cos 60^\circ = \frac{1}{2}$

$$(x-1)^2 = 4$$

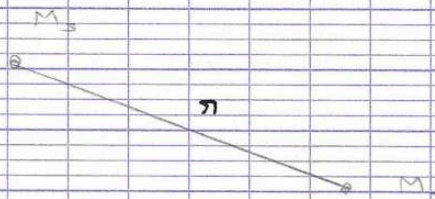
$$(x-1) = \pm 2$$

$$\Rightarrow x = 1 \pm 2$$

Donc $x =$ $\begin{matrix} x = -1 \\ x = 3 \end{matrix}$

Mouvement dans un champ de force en $1/R^2$

Exercice 30



TERRE soumise à une énergie potentielle $\bar{E}_p$:

$$\bar{E}_p = - \frac{GM_S M_T}{r}$$

$$G = 6,67 \cdot 10^{-11}$$

1) $\vec{a}_0$ est une constante du mouvement

$$\vec{a}_0 = \vec{OM} \wedge M_T \vec{v}$$

Montrons que $\frac{d\vec{a}_0}{dt} = \vec{0}$

$$\begin{aligned} \frac{d\vec{a}_0}{dt} &= \frac{d}{dt} (\vec{OM} \wedge M_T \vec{v}) = \frac{d\vec{OM}}{dt} \wedge M_T \vec{v} + \vec{OM} \wedge M_T \frac{d\vec{v}}{dt} \\ &= \vec{v} \wedge M_T \vec{v} + \vec{OM} \wedge M_T \frac{d\vec{v}}{dt} \end{aligned}$$

$$= \vec{OM} \wedge m_r \frac{d\vec{v}}{dt} = \vec{OM} \wedge m_r \vec{a}$$

$$\frac{d}{dt} (\vec{A} \wedge \vec{B}) = \frac{d\vec{A}}{dt} \wedge \vec{B} + \vec{A} \wedge \frac{d\vec{B}}{dt}$$

$$\frac{d\vec{\sigma}_0}{dt} = \vec{OM} \wedge \vec{F} \quad \text{OR} \quad \vec{F} \text{ est une force centrale}$$

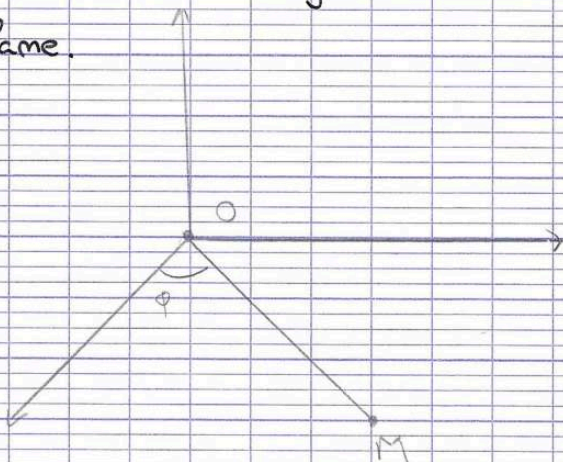
$\Rightarrow \vec{OM}$ et $\vec{F}$ ont donc la même direction

$$\frac{d\vec{\sigma}_0}{dt} = 0 \Rightarrow \vec{\sigma}_0 = \text{cte}$$

On sait que $\vec{\sigma}_0$ est $\perp$ à $\vec{OM}$ et $\vec{v}$

$\Rightarrow \vec{\sigma}_0$ est $\perp$ au plan contenant les deux vecteurs $(\vec{OM}, \vec{v})$

donc $\vec{\sigma}_0$ garde une direction fixe dans l'espace
Le plan $(\vec{OM}, \vec{v}) \perp$ à $\vec{\sigma}_0$ correspond au plan contenant la trajectoire de M. Donc la trajectoire est plane.



2)

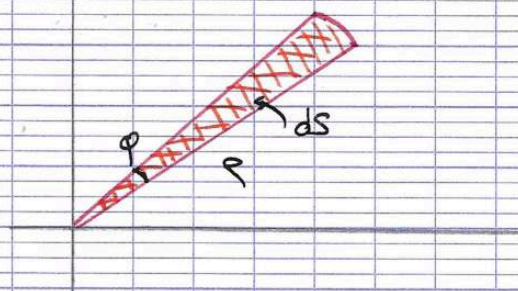
$$\vec{OM} = r \vec{e}_r$$

$$\vec{v} = \frac{d\vec{OM}}{dt} = \dot{r} \vec{e}_r + r \dot{\phi} \vec{e}_\phi$$

$$\vec{\sigma}_0 = m_r r \vec{e}_r \wedge (\dot{r} \vec{e}_r + r \dot{\phi} \vec{e}_\phi)$$

$$= m_r r \dot{\phi} (\vec{e}_r \wedge \vec{e}_\phi) + m_r r^2 \dot{\phi} (\vec{e}_r \wedge \vec{e}_r)$$

$$\vec{\sigma}_0 = m_r r^2 \dot{\phi} \vec{e}_z$$



$$ds = \frac{r^2 d\phi}{2} \Rightarrow \frac{ds}{dt} = \frac{r^2}{2} \frac{d\phi}{dt} = \frac{1}{2} r^2 \dot{\phi} = \frac{\sigma}{2}$$

On a vu que $\sigma_0 = M_T r^2 \dot{\phi} = cte$
 $\Rightarrow r^2 \dot{\phi} = cte$

3) $\bar{L}_m = \frac{1}{2} M_T r^2 + \bar{L}_{pot}(r)$

$$\begin{aligned} \bar{L}_m &= \bar{L}_c + \bar{L}_p = \frac{1}{2} M_T v^2 - \frac{GM_S M_T}{r} \\ &= \frac{1}{2} M_T (\dot{r}^2 + (r\dot{\phi})^2) - \frac{GM_S M_T}{r} \\ &= \frac{1}{2} M_T \dot{r}^2 + \frac{1}{2} M_T r^2 \dot{\phi}^2 - \frac{GM_S M_T}{r} \end{aligned}$$

$$\begin{aligned} \sigma_0 &= M_T r^2 \dot{\phi} \\ \Rightarrow \dot{\phi} &= \frac{\sigma_0}{M_T r^2} \end{aligned}$$

$$\begin{aligned} \bar{L}_m &= \frac{1}{2} M_T \dot{r}^2 + \frac{1}{2} M_T r^2 \frac{\sigma_0^2}{M_T^2 r^4} - \frac{GM_T M_S}{r} \\ &= \frac{1}{2} M_T \dot{r}^2 + \frac{1}{2} \frac{\sigma_0^2}{M_T r^2} - \frac{GM_T M_S}{r} \end{aligned}$$

4) L'équa d'eq du mv+

Dans un système si $\bar{L}_m$ est une cte du mv+ alors: $\frac{d\bar{L}_m}{dt} = 0$

$\varphi \Rightarrow \tilde{L}_m$ s'exprime en coordonnées paramétriques
 $r(\varphi)$ permet de définir la trajectoire du mobile

$$u = \frac{1}{r} \quad u(\varphi) = \frac{1}{r} \Rightarrow r = \frac{1}{u}$$

$$\tilde{L}_m = \frac{1}{2} M_T \dot{r}^2 + \frac{\sigma_0^2}{2M_T} u^2 - GM_T M_S u$$

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{du} \frac{du}{dt} = \frac{dr}{d\varphi} \frac{du}{d\varphi} \frac{d\varphi}{dt}$$

$$= -\frac{1}{u^2} \frac{du}{d\varphi} \dot{\varphi}$$

$$= -\frac{1}{u^2} \frac{du}{d\varphi} \frac{\sigma_0}{M_T} u^2$$

$$\dot{r} = -\frac{\sigma_0}{M_T} \frac{du}{d\varphi}$$

$$\tilde{L}_m = \frac{1}{2} M_T \left(-\frac{\sigma_0}{M_T} \frac{du}{d\varphi} \right)^2 + \frac{\sigma_0^2}{2M_T} u^2 - GM_T M_S u$$

$$= \frac{\sigma_0^2}{2M_T} \left[\left(\frac{du}{d\varphi} \right)^2 + u^2 \right] - GM_T M_S u$$

$$= \frac{\sigma_0^2}{2M_T} \left[2 \frac{du}{d\varphi} \frac{d^2 u}{d\varphi^2} + 2u \frac{du}{d\varphi} \right] - GM_T M_S \frac{du}{d\varphi} = 0$$

$$\Leftrightarrow \left(\frac{\sigma_0^2}{2M_T} \left[\frac{du}{d\varphi} + u \right] - GM_T M_S \right) \frac{du}{d\varphi} = 0$$

Soit $\frac{du}{d\varphi} = 0 \Rightarrow u$ ne dépend pas de φ

$$\Leftrightarrow \frac{d^2 u}{d\varphi^2} + u = \frac{GM_T M_S}{\sigma_0^2} \quad \text{on pose } p = \frac{\sigma_0^2}{GM_T M_S}$$

$$\frac{d^2 u}{d\varphi^2} + u = \frac{1}{p}$$

homogène: $\frac{d^2 u}{d\varphi^2} + u = 0 \quad u(\varphi) = A \cos(\varphi + \phi)$
 avec les conditions initiales $\phi = 0$

générale: $u = A \cos \varphi + \frac{1}{p} = \frac{A p \cos \varphi + 1}{p}$

$$p = \frac{p}{1 + A p \cos \varphi} = \frac{p}{1 + e \cos \varphi}$$